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## **METHODS APPLIED TO AIRCRAFT IDENTIFICATION**

### **System identification**

System identification is the process of determining an adequate mathematical model on the basis of input-output data.

The aim of identification is numerical values calculations of mathematical model parameters of the process or object on the basis of input and output signals gathered during identification experiment.

The identification task is to estimate process parameters. The parameter estimation is experimental determination of parameters values which direct dynamic behavior of the process when the structure of the model is known [4].

System identification consists the following steps:

- model type and structure postulation
- model parameter estimation method determination
- identification experiment design (input and output data determination, input function selection, measurement methodology determination)
- experiment realization and data logging
- model parameter calculations
- results analysis and verification [6].

There are two major identification types: on-line and off-line identification. On-line identification is operated during the process parameters of which are to be identified. In this case in every time step parameters are estimated so the changes during the process are taken into account in identification. This type of identification is called adaptive or recurrent identification. In case of the first one model structure can be changed but in the second one, the model structure must be determined before the identification and it remains during the whole process. Both of this identification types can lead to adaptive control which respond to the process dynamics. Off-line identification makes the parameters values calculation on the basis of previously gathered data so it can be used in control with model of the system but it cannot react to the changes in the process.

In respect to identification experiment identification can be active or passive. In case of active identification the experiment is planned and determined input signal is provided for the object. Passive identification is proceeding during the system exploitation [7].

There are different approaches to process identification result from process or object inside the structure knowledge. First approach is identification

without using the physical model of the process and equations describing proceeding phenomena. In this case input and output signals are assumed to be known but identified process or object is treated as so called black box. Identification without interfere in process structure is called black-box identification. But if during the identification there are used laws ruling considered process or the principle of work of the system and on this basis the physical and mathematical model are determined, then the grey-box identification is used [6].

Identification is possible in time and frequency domain. In this paper the time domain is discussed.

### System models

A real system can be described with the assistance of physical, mathematical and simulation models. The physical model of considered system is its idealization by replacement real system elements by ideal ones. Constrained assumptions are given and insignificant for considered phase of research features are neglected. The physical model should return the most important features of the real system. The mathematical model is the physical system description using mathematical language. If the mathematical model is written in program form, then it is called simulation model.

Using different criterion following types of models can be listed: static and dynamic models, linear and nonlinear, continuous and discrete. Dynamical model reflect system dynamical properties and changes in considered system proceed in time. Static model examine considered object in steady state that means assuming external influence is stable in time. In case of continuous models it is assumed that time is changing in continuous way so the all values of time derivative set is uncountable. Model parameters values are determined in optional time moments. Continuous model is mathematically described by differential and partial differential equations. In discrete models time has only discriminated values so the all values of time derivative set is countable. Model parameters values are determined in determined discrete time moments. Discrete model is described by difference equations. Nonlinear models are described by nonlinear differential or difference equations while linear models are in form of linear difference or differential equations systems and linear algebraic equations. Very often nonlinear models are simplified to linear ones because large number of identification methods can be used for linear models only [3], [4], [8].

**Table 1.**

<b>AR model (AutoRegressive model )</b>	
$\hat{y}_{AR}(k) = \mathbf{v}_{AR}(k)\boldsymbol{\theta}_{AR}$	
$\mathbf{v}_{AR}(k) = [-y(k-1), -y(k-2), \dots, -y(k-n)]$	$\boldsymbol{\theta}_{AR} = [a'_1, a'_2, \dots, a'_n]'$

<b>ARX model</b> (AutoRegressive model with eXogenous input)
$\hat{\mathbf{y}}_{\text{ARX}}(k) = \mathbf{v}_{\text{ARX}}(k)\boldsymbol{\theta}_{\text{ARX}}$
$\mathbf{v}_{\text{ARX}}(k) = [-y(k-1), \dots, -y(k-n), \hat{\eta}(k-1), \dots, \hat{\eta}(k-n)],$ $\boldsymbol{\theta}_{\text{ARX}} = [a'_1, \dots, a'_n, c'_1, \dots, c'_n]'$
<b>MA model</b> (Moving Average model) ( <b>FIR</b> )
$\hat{\mathbf{y}}_{\text{MA}}(k) = \mathbf{v}_{\text{MA}}(k)\boldsymbol{\theta}_{\text{MA}}$
$\mathbf{v}_{\text{MA}}(k) = [u(k-d), \dots, u(k-n-d)], \quad \boldsymbol{\theta}_{\text{MA}} = [b'_0, \dots, b'_n]'$
<b>MAX model</b> (Moving Average model with eXogenous input)
$\hat{\mathbf{y}}_{\text{MAX}}(k) = \mathbf{v}_{\text{MAX}}(k)\boldsymbol{\theta}_{\text{MAX}}$
$\mathbf{v}_{\text{MAX}}(k) = [u(k-d), \dots, u(k-n-d), \hat{\eta}(k-1), \dots, \hat{\eta}(k-n)],$ $\boldsymbol{\theta}_{\text{MAX}} = [b'_0, \dots, b'_n, c'_1, \dots, c'_n]'$
<b>ARMA model</b> (AutoRegressive Moving Average model)
$\hat{\mathbf{y}}_{\text{ARMA}}(k) = \mathbf{v}_{\text{ARMA}}(k)\boldsymbol{\theta}_{\text{ARMA}}$
$\mathbf{v}_{\text{ARMA}}(k) = [u(k-d), \dots, u(k-n-d), -y(k-1), -y(k-2), \dots, -y(k-n)]$ $\boldsymbol{\theta}_{\text{ARMA}} = [b'_0, \dots, b'_n, a'_1, a'_2, \dots, a'_n]'$
<b>ARMAX model</b> (AutoRegressive Moving Average model with eXogenous input)
$\hat{\mathbf{y}}_{\text{ARMAX}}(k) = \mathbf{v}_{\text{ARMAX}}(k)\boldsymbol{\theta}_{\text{ARMAX}}$
$\mathbf{v}_{\text{ARMAX}}(k) = [u(k-d), \dots, u(k-n-d), -y(k-1), \dots, -y(k-n), \hat{\eta}(k-1), \dots, \hat{\eta}(k-n)]$ $\boldsymbol{\theta}_{\text{ARMAX}} = [b'_0, \dots, b'_n, a'_1, \dots, a'_n, c'_1, \dots, c'_n]'$
where: $\mathbf{v}(k)$ – input vector; $\boldsymbol{\theta}$ – coefficients vector; $\hat{\mathbf{y}}(k)$ – output vector; $b'_0, \dots, b'_n$ ; $a'_1, \dots, a'_n$ ; $c'_1, \dots, c'_n$ – weighting coefficient; $n$ – model order (number of time moments in which input signal was measured); $d$ – delay discrete value.

Linear model assumes linear dependence between input signal  $\mathbf{v}(k)$  and output signal  $y(k)$  in  $k$  moment. When some process is considered, input signal is a vector of the following form:

$$\mathbf{v}(k) = [v_1(k), v_2(k), \dots, v_m(k)]. \quad (1)$$

For every following vector term there is corresponding coefficient from coefficients vector  $\boldsymbol{\theta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_m]$ . Occurring noises, which values are not known, are taken into consideration by  $\eta(k)$  element.

$$\begin{aligned} \mathbf{y}(k) &= \vartheta_1 v_1(k) + \vartheta_2 v_2(k) + \dots + \vartheta_m v_m(k) + \eta(k) = \\ &= \sum_{i=1}^m \vartheta_i v_i + \eta(k) = \mathbf{v}(k)\boldsymbol{\theta} + \boldsymbol{\eta}(k) \end{aligned} \quad (2)$$

$$\mathbf{v}(k) = [v_1(k), v_2(k), \dots, v_m(k)] \quad \boldsymbol{\theta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_m]^T. \quad (3)$$

Coefficients vector  $\boldsymbol{\theta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_m]^T$  is not known and determining of it is the identification aim. Coefficients values determined for the model  $\hat{\boldsymbol{\theta}} = [\hat{\vartheta}_1, \hat{\vartheta}_2, \dots, \hat{\vartheta}_m]^T$  differs from these from the real process. These differences are named model error and it is described as the difference between process real and model output signal:

$$\mathbf{e}(k) = \mathbf{y}(k) - \hat{\mathbf{y}}(k) = \mathbf{y}(k) - \mathbf{v}(k)\hat{\boldsymbol{\theta}}. \quad (4)$$

Most commonly used are so called regression models. In Table 1 there are gathered input vectors  $\mathbf{v}(k)$ , coefficients vectors  $\boldsymbol{\theta}$  and output vectors  $\hat{\mathbf{y}}(k)$  of mostly used linear regression models [4].

### Aircraft model

Another type of models besides regression ones, are state-space models of the following form [2]:

– state equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (5)$$

– output equation:

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t), \quad (6)$$

where:  $\mathbf{x}(t)$  – vector of state variables,

$\mathbf{y}(t)$  – vector of output variables,

$\mathbf{A}$  – the state matrix,

$\mathbf{B}$  – the input matrix,

$\mathbf{C}$  – the output matrix,

$\mathbf{D}$  – the direct matrix.

When airplane systems are considered, the output variables are assumed to be the state ones and that leads to simplification:  $\mathbf{y}(t) = \mathbf{x}(t)$ .

In literature, motion of the airplane is treated in simplified way as two separate cases: longitudinal and lateral motions. The longitudinal motion may be described by state equation of the form:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_u & x_w & x_q & x_\theta \\ z_u & z_w & z_q & z_\theta \\ m_u & m_w & m_q & m_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} x_\eta & x_\tau \\ x_\eta & x_\tau \\ x_\eta & x_\tau \\ x_\eta & x_\tau \end{bmatrix} \begin{bmatrix} \eta \\ \tau \end{bmatrix} \quad (7)$$

while the lateral-directional state equation is :

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} y_v & y_p & y_r & y_\phi \\ l_v & l_p & l_r & l_\phi \\ n_v & n_p & n_r & n_\phi \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} y_\xi & y_\zeta \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \quad (8)$$

where:  $x, y, z$  – axial drag force, side force, normal lift force components,  
 $l, m, n$  – rolling, pitching, yawing moment components,  
 $p, q, r$  – roll, pitch, yaw rate,  
 $u, v, w$  – axial, lateral, normal velocity components,  
 $\phi, \theta$  – roll and pitch angle respectively,  
 $\eta, \xi, \zeta, \tau$  – elevator, aileron, rudder displacement and engine thrust.

### Identification methods

In the paper identification methods commonly used for aircraft identification in the time domain are discussed. The most general is Maximum Likelihood method (ML). It uses the following description of the system:

$$\dot{x}(t) = f[x(t), u(t), \beta], \quad x(t_0) = x_0, \quad (9)$$

$$y(t) = g[x(t), u(t), \beta]. \quad (10)$$

Output equation:

$$z(t_k) = y(t_k) + G \cdot v(t_k), \quad (11)$$

where:  $x$  –  $n_x \times 1$  column vector of state variables,

$u$  –  $n_u \times 1$  control input vector,

$y$  –  $n_y \times 1$  system output vector,

$\beta$  –  $n_q \times 1$  vector of system parameters,

$f, g$  – general nonlinear real-valued functions,

$z(t_k)$  – discrete measurements of the model output  $y(t_k)$ ,

$k$  – discrete time index,

$v(t_k)$  –  $n_v \times 1$  measurement noise vector.

In ML method it is assumed that the measurement noise is characterized by a sequence of independent Gaussian random variables with zero mean and identity covariance. The elements  $\lambda$  of process noise distribution matrix  $F$  and the initial conditions  $x_0$  are unknown. Also  $G$  (measurement noise distribution matrix) is unknown. Searched unknown parameter vector is:

$$\Theta = [\beta^T \lambda^T x_0^T]^T. \quad (12)$$

In case of aircraft identification there are the following assumptions:

- the input signal  $u(t_k)$  is independent of the output signal,
- the measurement errors  $v(t_k) = z(t_k) - y(t_k)$  at different time points are statistically independent and they are assumed to be distributed with zero mean and covariance matrix  $R$ :

$$E\{v(t_k)\} = 0; \quad E\{v(t_k)v^T(t_l)\} = R \cdot \delta_{kl}, \quad (13)$$

- there is only measurement noise in the system, the process noise is neglected [5].

ML estimates are obtained by minimization of the negative logarithm of the likelihood function called cost function  $J(\Theta, R)$ :

$$J(\Theta, R) = \frac{1}{2} \sum_{k=1}^N [z(t_k) - y(t_k)]^T R^{-1} [z(t_k) - y(t_k)] + \frac{N}{2} \ln[\det(R)] + \frac{Nn_y}{2} \ln(2\pi). \quad (14)$$

When measurement error covariance matrix  $R$  is known, the cost function reduces to:

$$J(\Theta) = \frac{1}{2} \sum_{k=1}^N [z(t_k) - y(t_k)]^T R^{-1} [z(t_k) - y(t_k)]. \quad (15)$$

As an optimization method the Gauss-Newton method and modifications Levenberg-Marquardt method are commonly used in case of aircraft parameter identification [5], [6].

One of ML method simplifications is called Equation Error Method (Regression Method) which consists all types of Least Squares Techniques. They minimize a cost function defined directly in terms of an input-output equation. The cost function is based on matrix algebra operations. In identification of 6 DOF object, especially aircrafts, the most commonly used are Ordinary Least Squares, Weighted Least Squares and Total Least Squares methods. In OLS the independent variables are assumed to be without error and noise while dependent variables have uniformly distributed noise. WLS method is an extension of

LS which takes into consideration and explains residuals. But one of its disadvantages is that while measurements errors and noise in the independent variables are present, the WLS method yields asymptotically biased and inconsistent estimates. Another extension of LS is Total LS method which deals with noise in the independent variables [5].

First, the general Least Squares idea is briefly discussed. The error can be obtained also in matrix form:

$$\varepsilon = Y - X\theta. \quad (16)$$

By minimizing the sum of the residuals squares, the LS estimates of the unknown parameters  $\theta$  are obtained. The cost function is defined as:

$$\begin{aligned} J(\theta) &= \frac{1}{2} \sum_{k=1}^N \varepsilon^2(k) = \frac{1}{2} \varepsilon^T \varepsilon = \frac{1}{2} [Y - X\theta]^T [Y - X\theta] = \\ &= \frac{1}{2} [Y^T - \theta^T X^T] [Y - X\theta]. \end{aligned} \quad (17)$$

The minimum of the cost function  $J(\theta)$  is obtained by setting the gradient of  $J(\theta)$  with respect to  $\theta$  to zero because the error  $\varepsilon(k)$  is a linear function.

$$\frac{\partial J(\theta)}{\partial \theta} = -Y^T X + \theta^T (X^T X) \quad \wedge \quad \frac{\partial J(\theta)}{\partial \theta} = 0 \quad \Rightarrow \quad (X^T X)^T \hat{\theta} = X^T Y. \quad (18)$$

The estimated parameters are calculated from:

$$\hat{\theta} = (X^T X)^{-1} X^T Y. \quad (19)$$

The variation of LS is Weighted Least Squares method, where matrix with weights  $W$  is added. Instead of minimizing the error  $\varepsilon^T \varepsilon$  like in LS, the  $\varepsilon^T W \varepsilon$  factor is minimized. After this change, the cost function is:

$$\begin{aligned} J(\theta) &= \frac{1}{2} \sum_{k=1}^N w_k \varepsilon^2(k) = \frac{1}{2} \varepsilon^T W \varepsilon = \frac{1}{2} [Y - X\theta]^T W [Y - X\theta] = \\ &= \frac{1}{2} [Y^T - \theta^T X^T] [Y - X\theta]. \end{aligned} \quad (20)$$

The estimates are obtained from:

$$\hat{\theta} = (X^T W X)^{-1} X^T W Y. \quad (21)$$

The Total Least Squares method allows the noise in the independent variables to be accounted for. It is very strong advantage in comparison to LS which yields biased estimates in the presence of systematic errors and noise in the independent variables.

The equation (16) in TLS method is rewritten as:

$$Y = (X - \mu)\theta + \varepsilon \quad (22)$$

and after few transformations as:

$$\begin{bmatrix} X & Y \end{bmatrix} - [\mu \quad \varepsilon] \begin{bmatrix} \theta \\ -1 \end{bmatrix} = \begin{bmatrix} X & -\tilde{\Delta} \end{bmatrix} \begin{bmatrix} \theta \\ -1 \end{bmatrix} = 0, \quad (23)$$

where:  $\tilde{X} = [X \quad Y]$  – the compounded data matrix of size  $(N \times n_q + 1)$ ,

$\tilde{\Delta} = [\mu \quad \varepsilon]$  – the compounded noise vector.

The compounded matrix can be written in following way:

$$\begin{bmatrix} X & Y \end{bmatrix} = U \Sigma V^T, \quad (24)$$

where:  $\Sigma = [\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_{n_q+1})]$  – diagonal matrix of singular values such that  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{n_q+1}$ ;

$U, V$  – the left and right singular matrices.

The smallest of the singular values  $(\sigma_1, \sigma_2, \dots, \sigma_{n_q+1})$  which is  $n_q + 1$  the one, corresponds to the minimum. The solution is obtained from the last column of  $V$ , which corresponds to the smallest singular value:

$$\theta = -\frac{v}{\lambda}, \quad (25)$$

where:  $\lambda$  – the last element;

$v$  – the vector of the first  $n_q$  elements of the last column of  $V$ .

Another LS type technique is Instrumental Variable method (IV). Introduced so-called instrument variables are responsible for canceling out the effect of correlated noise.  $z(k)$  can be strongly correlated with the independent variables  $x(k)$  but it cannot be correlated with an equation error  $\varepsilon(k)$ . That leads to the statement that  $E\{z(k)x^T(k)\}$  is a positive definite matrix and

$$E\{z(k)\varepsilon(k)\} = 0. \quad (26)$$

The estimation parameters are obtained from the equation:

$$\hat{\theta} = (Z^T X)^{-1} Z^T Y, \quad (27)$$

where  $Z$  – instrumental variables matrix.

The main difficulties in case of this method is finding an appropriate set of such variable  $Z$ .

Many of EEMs can be used for real-time identification after some simplifications and assumptions. Adaptive algorithms that are based on the EEM and



can be easily implemented for real-time model parameter determination are called Recursive Methods (RM). They utilize the data point-by-point as they become available. They cater for systems with time-varying parameters. Requirements of computer memory are small when we deal with these methods because storage of past data is not required. That's make them suitable for online implementation even on a small onboard computer. But they have also some limitations. Unfortunately there may be problem with slow convergence of standard RPE methods (ex. RLS –Recursive Last Squares method) which may not be adequate for real-time fault detection or to detect sudden changes in dynamics. The convergence can be improved by incorporating a forgetting factor to discard older data but it would cause increased noise sensitivity. Although a wrong choice of this factor results in estimates oscillating around the true values. Also problems may occurs because of lack of or limited information content pertaining to dynamic motion for example control and motion variables could be below the noise level during steady level flight. RPE methods do not verify data colinearity [5], [6], [7].

The RLS algorithm can be summed up as:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K(k+1) \left[ y(k+1) - x^T(k+1)\hat{\theta}(k) \right]; \quad (28)$$

$$K(k+1) = \frac{P(k)x(k+1)}{1 + x^T(k+1)P(k)x(k+1)}; \quad (29)$$

$$P(k+1) = P(k) - K(k+1)x^T(k+1)P(k). \quad (30)$$

If there is a need of faster adaptation with time after using RLS method, another possibility is Recursive Weighted Least Squared, in which past information are quickly discarded. Forgetting factor  $\lambda$ , which introduces an exponentially decaying weights on the past measurements, is used in such cases. Smaller values of  $\lambda$  neglect more and more data points that are from the past. The minimized cost function is described as:

$$J(\theta) = \frac{1}{2} \sum_{i=1}^k \lambda^{k-i} \varepsilon^2(i). \quad (31)$$

Other real-time methods are Filtering Identification Methods (FIM). The most commonly used is Kalman Filter (KF) for linear system and Extended Kalman Filter (EKF) for nonlinear ones. In Kalman Filter the linear system of followed form is considered [1]:

$$\mathbf{x}_k = \mathbf{F} \cdot \mathbf{x}_{k-1} + \mathbf{v}_{k-1}, \quad \mathbf{z}_k = \mathbf{H} \cdot \mathbf{x}_k + \mathbf{w}_k, \quad (32)$$

where:  $\mathbf{F}$  – state matrix, the same for every step ( $k$ ),  $\mathbf{H}$ -observation matrix; constant for every step ( $k$ ),  $\mathbf{v}_k$ ,  $\mathbf{w}_k$  – noise vectors of white-noise with zero average and covariance matrixes  $\mathbf{Q}_k$ ,  $\mathbf{R}_k$ .

The algorithm consist two stages: prediction and actualization. At the beginning, starting values are assumed (state vector  $\hat{\mathbf{x}}_0$  and error covariance matrix  $\mathbf{P}_0$ ) for time value  $t_0$ .

Prediction stage consists:

- (1) Next step state vector prediction:

$$\hat{\mathbf{x}}_{k+1}^- = \mathbf{F} \hat{\mathbf{x}}_k. \quad (33)$$

- (2) Next step error covariance matrix of state vector prediction:

$$\mathbf{P}_{k+1}^- = \mathbf{F} \mathbf{P}_k \mathbf{F}^T + \mathbf{Q}_k. \quad (34)$$

Actualization stage consists steps:

- (1) Kalman gain matrix calculations:

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_k^- \mathbf{H}^T + \mathbf{R}_k)^{-1}. \quad (35)$$

- (2) Predicted in step (1) state vector actualization on the basis of current measurement data:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H} \hat{\mathbf{x}}_k^-). \quad (36)$$

- (3) Predicted in step (2) state error covariance matrix actualization:

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_k^-. \quad (37)$$

KF is easy to implement. During the calculations state vector accuracy is estimated ( $\mathbf{P}_k$  covariance matrix). The major disadvantage is implementation to linear systems only. There may be also difficulties with estimation of state vector error covariance matrix  $\mathbf{Q}_k$  before starting the algorithm. Important limitation result from the assumption of white-noise in the system.

In case of EKF, nonlinear systems may be identified, which is the major advantage of this method but there is a necessity of functions from state and observation equations linearization to obtain Jacobi matrix. Calculations of this matrix are complicating the algorithm [1], [3].

There are many more methods in the time domain that can be used for identification for example all kinds of Neural Networks algorithms but they were not discussed in this paper.

## Conclusions

In the paper the system identification is discussed first as a general matter. Different types of identification are presented and the classification is made. Then models of systems in general form are described, both regression and state-space kind. As an example of state-space model the simplified 6DOF air-

craft model is proposed to further identification. Then identification methods like ML, Recursive and Filtering methods are considered for parameter estimation. Further research will consist comparison of real-time method which can deal with state-space form of aircraft model and implementation in control system based on adaptive identification.

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## **МОДИФІКАЦІЯ ФІЛЬТРІВ ОБРОБКИ БАГАТОСПЕКТРАЛЬНИХ ЗОБРАЖЕНЬ ДЛЯ ЗАДАЧ ДИСТАНЦІЙНОГО ЗОНДУВАННЯ ЗЕМЛІ**

### **Вступ**

Методи дистанційного зондування Землі (ДЗЗ) широко використовуються для розв'язання різноманітних задач [1]. Нині у світі є понад 20 компаній, які займаються обробкою зображень ДЗЗ, серед них: Jeppesen, Trimble, ArcGis, GeoGude, Spectra Precision та інші.

Сучасні апаратні засоби дистанційного зондування Землі з великовисотних носіїв фіксують зображення у видимому та інфрачервоному діапазонах. Сукупність розподілів яскравості відповідних зображень утворює багатовимірний геометричний об'єкт – багатоспектральне зображення